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The impact of forward trading on the spot power price volatility with Cournot competition

Sandro Sapio*
Agnieszka Wyłomańska**

* Department of Economic Studies, University of Naples Parthenope, Italy
** Hugo Steinhaus Center, Wrocław University of Technology, Poland

Hugo Steinhaus Center
Wrocław University of Technology
Wyb. Wyspiańskiego 27, 50-370 Wrocław, Poland
http://www.im.pwr.wroc.pl/~hugo/

The Impact of Forward Trading on the Spot Power Price Volatility with Cournot Competition

Sandro Sapio[1], Agnieszka Wylomanska[2]

[1] University of Naples Parthenope
Department of Economic Studies
Via Medina, nº 40
80133 Naples, Italy
Phone: +39-081-5474910
E-mail: alessandro.sapio@uniparthenope.it

[2] Wroclaw University of Technology
Institute of Mathematics and Computer Science
Janiszewskiego , nº 14a
50-370 Wroclaw, Poland
Phone: +48-071-3203183
E-mail: Agnieszka.Wylomanska@pwr.wroc.pl

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Abstract-**In this paper, we analyze the influence of forward trading on the volatility of spot power prices, in models where forward contracts are strategic tools used by energy producers to obtain profit security. We define volatility as the variance of the percentage change in spot power prices over a given time interval. As shown in Sapio (2008), volatility is related to stochastic fluctuations in preference and technology fundamentals, and is tuned by the price-elasticity of demand and supply, evaluated at equilibrium.**

We study two cases. First, we analyze the volatility implications of a model wherein the amount of forward trading is fixed, and producers compete *a la* Cournot. Fixed forward trading increases spot volatility, because forwards lower the spot price level, corresponding to a less elastic region of a linear demand function. However, if the amount of forward trading is endogenous, as in the two-stage model of Allaz (1992), producers can anticipate the spot market impact of stochastic shocks on fundamentals and ``sterilize'' them. As a result, spot price volatility is closer to the value implied by an efficient market. Our theoretical results are illustrated by means of a simple simulation study.

I. NOMENCLATURE

t: time index
a(t): maximum reservation price for electricity use (intercept of the demand function)
c(t): marginal cost of electricity generation
p(t): spot electricity price
q(t): spot electricity trading volume
n: number of electricity producers
F(t): forward electricity trading volume (sum over firms)
E[•|t-1]: conditional expected value operator
V[•]: variance operator

$$\xi_{px} \equiv \frac{\partial p(t)}{\partial x(t)}\frac{x(t)}{p(t)}$$: elasticity of the spot price with respect to the generic variable *x(t)*

$$\alpha(t) \equiv \frac{\dot{a}(t)}{a(t)}$$: growth rate of *a(t)*, i.i.d.

$$\gamma(t) \equiv \frac{\dot{c}(t)}{c(t)}$$: growth rate of *c(t)*, i.i.d.

$\dot{x}(t)$: the time derivative of the generic function $x(t)$.
Spot volatility: the square root of the variance of the spot

price growth rate over a given time interval.

II. INTRODUCTION

In recent years, the traditional regulatory framework for the wholesale electricity industry has been replaced in many countries by market competition. Electricity producers compete through both spot market bidding and forward trading. Albeit electricity users and producers are faced with extremely high risk (Eydeland and Wolyniec 2003, Geman 2005, Weron 2000, Weron 2006), hedging is not the only reason for the existence of forward markets (Allaz 1992). Forward contracts can be used by power suppliers strategically, in order to increase their spot market share. But if suppliers are allowed to trade forward, they can use information more efficiently, and once their forward market commitments are signed, they are ready to compete more aggressively on the spot market, enhancing competition. In turn, greater spot competition is believed to decrease the likelihood of spikes and to yield lower spot volatility.

In this paper, we reappraise the spot volatility impact of forward trading, based on a simple insight. Lower spot prices induced by forward trading may correspond to a less elastic region of the demand function –e.g. when demand is linear – possibly leading to higher spot volatility. Moreover, low volatility may be achieved even in tightly oligopolistic markets without forward trading, if markups are kept rather stable.

We analyze the impact of forward trading on spot volatility in a Cournot oligopoly, comparing two cases: fixed and endogenous forward coverage. Our analysis builds on the paper by Allaz (1992).

A forward contract is an agreement between two parties to buy or sell an asset at a pre-agreed future instant of time. Therefore, the trade date and delivery date are separated. They are used to control and hedge the market risk. If the transaction is collaterised, exchange of margin will take place according to a pre-agreed rule or schedule. Otherwise no asset of any kind actually changes hands, until the maturity of the contract. The forward price of such a contract is commonly contrasted with the spot price. The difference between the spot and the contract price is the contract premium or contract discount (Borgosz-Koczwara et al. 2007).

The paper is structured as follows. In Section III the spot volatility in models with fixed and endogenous and forward trading is analyzed. The results are illustrated by means of a simulation study.

III. MODELS WITH FORWARD TRADING

A. Fixed forward trading

The following analysis assumes symmetric Cournot competition in the spot electricity market, and lets the amount of forward contracts purchased by each of the n firms be fixed, or exogenously given, at the level f for individual firms, and at the market-wide level $F = n f \geq 0$ (see Niu, Baldick and Zhu 2005). Firms face an affine inverse demand function

$$p(t) = a(t) - q(t) \qquad (1)$$

Here, we assume that reservation prices, as measured by the intercept a(t), change over time, in response to events such as changes in weather, random shocks to preferences, and so forth. Conversely, we assume away any time variation in the slope of the demand function. Thus, for the sake of notational parsimony, we set the slope to 1.

Power generation companies are characterized by a linear cost function $C(t) = c(t)q(t)$, where $c(t)$ denotes the marginal cost of energy production, which is assumed constant across production levels, but variable over time. Indeed, this parameter captures the level of fuel costs, which fluctuate over time. Also, random shocks to the efficiency of power plants can give rise to fluctuations in $c(t)$ (see also the Nomenclature section at the beginning of the paper).

Firms choose their output so as to maximize profits:

$$\pi(t) = q(t)p(t) - C(t) + f(A - p(t)),$$

where A is the forward price. Taking account of all individual supply decisions, the expression for aggregate supply reads

$$q(t) = \frac{n[a(t) - c(t)] + F}{n+1} \qquad (2)$$

The market-clearing price is thus

$$p(t) = \frac{a(t) + nc(t) - F}{n+1} \qquad (3)$$

As in Sapio (2008), the variance of spot price rates of change reads

$$V_{fix}[\dot{p}(t)/p(t)] = \xi^2_{pa,fix} V[\alpha(t)] + \xi^2_{pc,fix} V[\gamma(t)] \qquad (4)$$

where

$$\xi_{pa,fix} = \frac{a(t)}{a(t) + nc(t) - F} \qquad (5)$$

and

$$\xi_{pc,fix} = \frac{nc(t)}{a(t) + nc(t) - F} \qquad (6)$$

Eq. (4) shows how the spot volatility depends on the volatility of fundamental drivers. Notice that, as n goes to infinity, the former (5) goes to zero and the latter (6) to 1. Hence, the spot price volatility approaches the volatility of marginal costs in a perfectly competitive market.

B. Endogenous forward trading

The reference model here is Allaz (1992), slightly extended to allow for uncertainty in marginal costs, in addition to the assumption of a random demand intercept as in the original model. We focus on the risk-neutral, Cournot conjectural variations case. The spot price implied by the Allaz model reads

$$p(t)=\frac{a(t)+n^2 c(t)}{n^2+1} \qquad (7)$$

This result is obtained by substituting the expression for the amount of forward trading

$$F(t-1)=\frac{n(n-1)}{n^2+1}E[a(t)-c(t)\mid t-1] \qquad (8)$$

(corresponding to Eq. 19 in Allaz 1992) into the spot price equation

$$p(t)=\frac{a(t)+nc(t)-F(t-1)}{n+1} \qquad (9)$$

under the assumption that producers are able to correctly guess the values of demand and cost parameters, using the information available up to time *t-1*:

$$E[x(t)\mid t-1]=x(t) \qquad (10)$$

with *x(t)* equal to either *a(t)* or *c(t)*. Eq. (7) implies that the spot volatility under endogenous choice of forward coverage, $V_{endo}[\dot{p}(t)/p(t)]$, depends on the elasticities

$$\xi_{pa,endo}=\frac{a(t)}{a(t)+n^2 c(t)} \qquad (11)$$

and

$$\xi_{pc,endo}=\frac{n^2 c(t)}{a(t)+n^2 c(t)} \qquad (12)$$

Here too, as n goes to infinity, the former elasticity (11) goes to zero, whereas the latter (12) goes to 1. Though, the volatility of spot prices with endogenous forward trading converges to the volatility of marginal costs faster than with fixed forwards, due to the terms in n squared.

C. The spot volatility impact of forward trading

As shown, the spot volatility does vary depending on whether forward coverage is fixed or endogenously determined, because the spot price responds in different ways to perturbations in cost and demand parameters. The information included in the previous sections can be useful in two respects. *First*, the results from the fixed forward model allow to study whether increasing the forward trading volume as such mitigates spot volatility. However, even beyond the impact of the sheer amount of forward trading, introducing forward contracts changes the behavior of electricity suppliers. As illustrated by the Allaz model, power producers can make strategic use of forward contracts, in order to improve their profit levels. Hence, a *second* goal of the analysis is to explore the issue of whether spot volatility is affected by the sequential strategies of electricity producers who trade both spot and forward. We pursue this goal by comparing the spot volatilities implied by fixed and endogenous forward trading models.

The relationship between spot volatility and the amount of forward trading is illustrated by (5) and (6), together with (4). Note that, because elasticities in (5) and (6) are increasing in F, forward trading increases the spot volatility by making the spot price more sensitive to fluctuations in both cost and demand parameters. This is because forwards lower the spot price level, corresponding to a less elastic region of the demand function, which is assumed linear. On these grounds, we conclude that *if forward trading is fixed, then greater forward coverage increases spot volatility.*

If the amount of forward trading is not fixed, suppliers can choose forward coverage strategically. To understand the impact of this, we wonder whether spot electricity markets with endogenous forward coverage are less volatile than with fixed forward trading. Formally, we ask whether

$$V_{fix} > V_{endo}$$

This is true if and only if

$$\Gamma < \frac{V[\alpha(t)]}{V[\gamma(t)]} \qquad (13)$$

with

$$\Gamma \equiv \frac{\xi^2_{pc,endo}-\xi^2_{pc,fix}}{\xi^2_{pa,fix}-\xi^2_{pa,endo}}$$ and $\xi^2_{pa,fix} > \xi^2_{pa,endo}$. from (5) and (11) if $n > 1$.

The threshold value is the ratio between two measures of relative price responsiveness. At the numerator, there is the difference between how responsive is price to marginal costs in the endogenous forwards case, and how responsive it is under the fixed forwards case. These responsiveness indicators are both increasing in n, because spot prices in a more competitive market provide more information on the underlying marginal costs, and therefore respond more to cost fluctuations. The former increases faster with n (compare (6) and (12)).

At the denominator, there appears the relative responsiveness of price to the demand variable *a*, under fixed and endogenous forward trading. Both are decreasing with *n* – i.e. higher when there is market power, because demand parameters enter the markups charged by suppliers. The latter decreases with *n* faster (compare (5) and (11)).

Condition (13) states that endogenous forward trading mitigates volatility more than fixed forward coverage, if the volatility of demand variables is greater than the volatility of marginal costs, according to a threshold value, dictated by the relative responsiveness of the price to the underlying demand and cost drivers.

Because price is more or less responsive to cost fluctuations depending on market power, it can be useful to assess how the impact of strategic trading on spot volatility is tuned by the strength of oligopolistic interactions and market power, which are inversely related to *n*. We thus study the behavior of Γ with respect to the number of firms, *n*, all else being given, to answer the question: is the volatility impact of endogenous forward trading different under different degrees of market competition?

Intuitively, as the market approaches perfect competition ($n\to\infty$), the spot price *p(t)* tends to the value of marginal costs *c(t)*, and accordingly, the spot price volatility tends to the volatility of production costs, $V[\gamma(t)]$. When producers are endowed with market power, instead, the spot price fluctuations depend also on the dynamics in the demand intercept *a(t)*, in that decision-making by suppliers needs to incorporate information regarding how users would respond to price policies. Barring perfect competition, the spot volatility typically lies between the volatility of demand and cost parameters; see also (4). Therefore, the spot market volatility can converge to the competitive value from "below" or from "above" as $n\to\infty$, depending on whether $V[\alpha(t)]$ or $V[\gamma(t)]$ is larger. In markets with very stable production costs and noisy demand, competition drives volatility down. The opposite holds when demand is stable and costs vary wildly.

This given, the impact of endogenous forward trading vis-à-vis fixed forwards can be understood by noting that, as indicated in sections *III.A* and *III.B*, the spot volatility converges faster to the volatility of marginal costs with strategically-driven forward coverage. Hence, as *n* increases, endogenous forwards boost volatility more if $V[\gamma(t)]$ is relatively large, but mitigate it more if $V[\gamma(t)]$ is relatively small. One way or the other, the volatility value under endogenous forwards is closer to what is implied by an efficient market, than under fixed forwards, for all values of *n*. This is consistent with the idea that, if suppliers are allowed to choose the amount of forward coverage, as in the Allaz model, they can use information more efficiently. However, the volatility implied by a competitive market might be very large, if marginal costs are very noisy.

To formally assess this intuition, let us study how the threshold Γ with respect to the number of firms *n*. Using (13), it is straightforward to verify that monopoly ($n=1$) is a sufficient condition for $V_{fix} > V_{endo}$ for all $F\geq 0$. Indeed, if $n=1$,

$$\Gamma = -\frac{c(t)^2}{a(t)^2} < 0 < \frac{V[\alpha(t)]}{V[\gamma(t)]}$$

This means that, if the market is fully concentrated, strategically-driven forward transactions are beneficial towards spot volatility reduction – consistent with Allaz (1992).

What is the behavior at larger values of *n*? Suppose increasing *n* causes Γ to increase. Then there may exist a value n^* such that (13) is no longer satisfied for all $n > n^*$. If so, allowing for endogenous forward trading in a market which is already rather competitive would actually make the spot market more volatile. Suppose instead that Γ is decreasing in *n*. In such a case, endogenous forward trading is most beneficial for volatility, regardless of the degree of market competition. The condition (13) is however rather cumbersome. This is why we resort to a simulation methodology, as in the following section.

D. A simulation study

In order to illustrate the theoretical results obtained in the previous sections, we consider the symmetric Cournot competition in the spot electricity market with fixed and endogenous forward trading. By means of simulations, we illustrate the behavior of spot volatility and the Γ coefficient with respect to *n*.

In our simulation study, we assume the sequences $\alpha(t)$ and $\gamma(t)$ are i.i.d. random variables with standard normal distribution mutually independent. In this case the sequences *a(t)* and *c(t)* are independent random walks given by

$$\log a(t) = \log a(t-1) + \alpha(t)$$
$$\log c(t) = \log c(t-1) + \gamma(t).$$

In Fig. 1 we illustrate the prices *p(t)* for three values of *n* in case of fixed forward trading (top panel) as well as the endogenous forward trading (bottom panel). Moreover, on both panels the star lines represent the sequence *c(t)*. It is worth pointing out that for a larger number of suppliers, the price tends to the marginal cost.

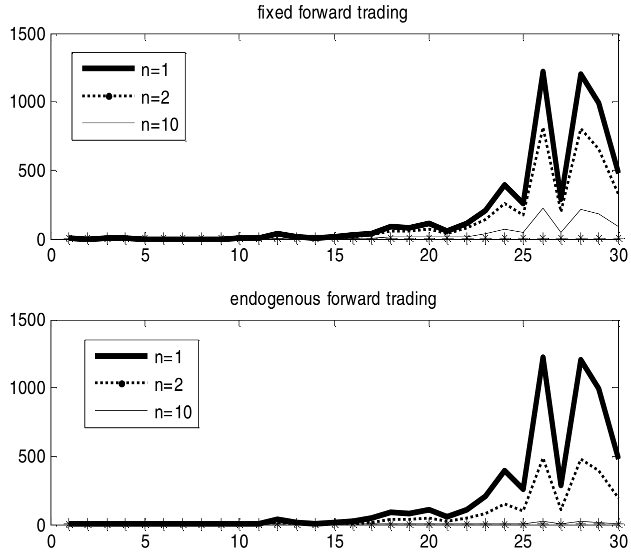

Fig.1. The prices *p(t)* for different values of *n*. On both panels the star lines represent the sequence *c(t)*.

In Fig. 2, 3 and 4 we show the behavior of spot price volatility for different numbers of suppliers in the two cases under consideration.

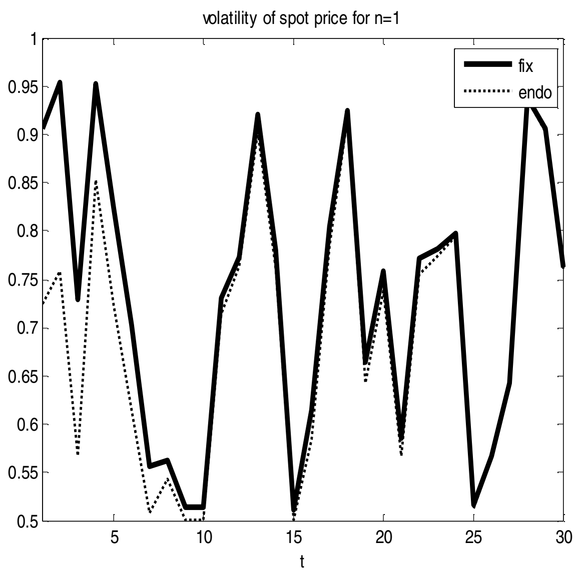

Fig.2. The volatility of spot price for *n=1*.

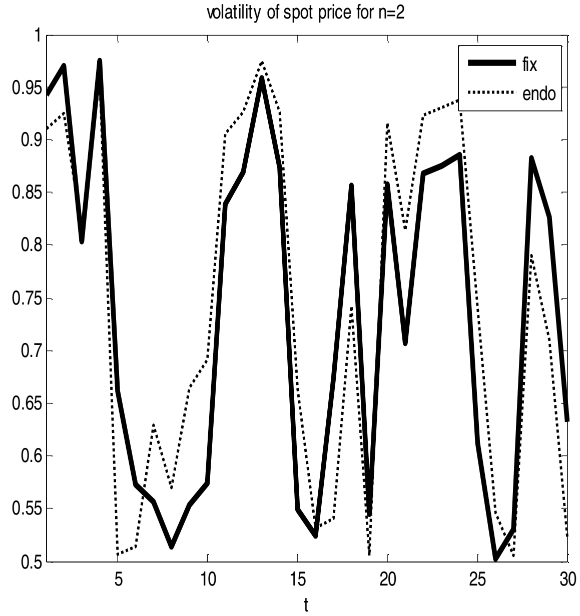

Fig.3. The volatility of spot price for *n=2*.

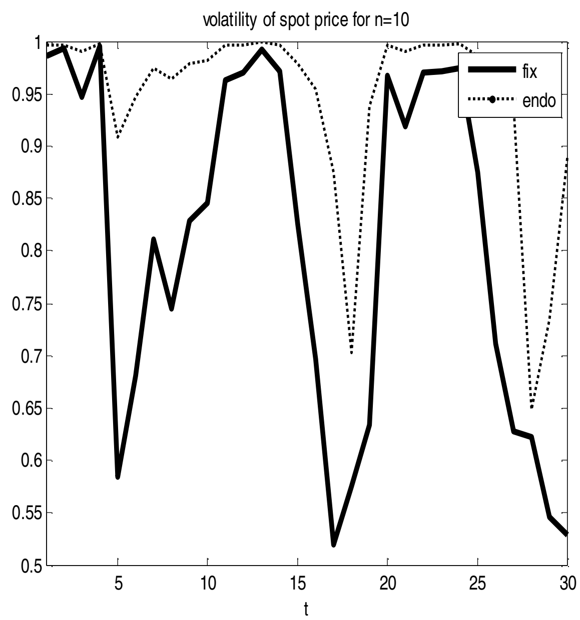

Fig.4. The volatility of spot price for *n=10*.

As we see, for *n=1* (the monopoly case) the volatility of spot prices in case of fixed forward trading exceeds the volatility in the endogenous forward case. This confirms the theoretical result obtained in section *III.C*. No clear ranking emerges between the fixed and the endogenous cases when *n=2,* whereas *n=10* implies that strategically-driven forward transactions make the spot market relatively more unstable.

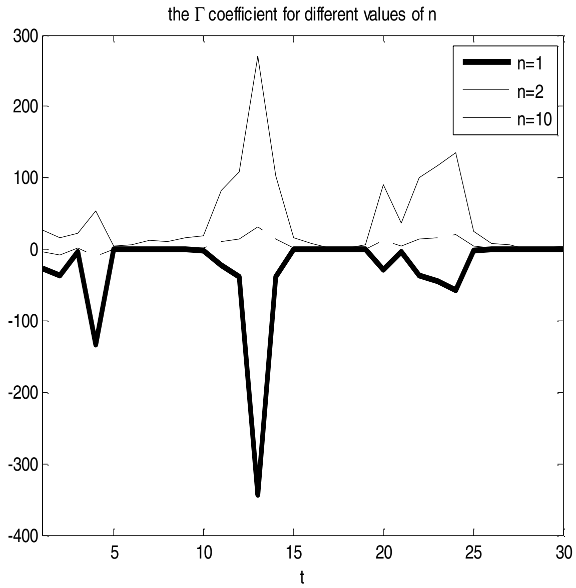

Fig.5. The Γ coefficient for different values of n.

In Fig. 5 we illustrate how the n parameter influences on the Γ coefficient. As before, we consider three values: $n=1$, 2 and 10. The Γ coefficient increases with respect to the number of suppliers, therefore the inequality given in (13) is less likely to hold for larger value of n. Thus, the parametrization used here (i.e. the given values of $V[\alpha(t)]$ and $V[\gamma(t)]$) implies that strategically-driven forward trading when the market is already rather competitive can magnify volatility. However, had we chosen smaller values of $V[\gamma(t)]$ and larger values of $V[\alpha(t)]$, introducing endogenous forward trading would actually *reduce* volatility.

The key is that endogenous forward trading pushes the price volatility closer to the volatility of marginal costs. But this may not be good news for the market, if marginal costs experience wide fluctuations.

IV. Conclusion

We have evaluated the price volatility in the Cournot model with fixed and endogenous forward trading, when suppliers use derivatives as components of their bidding strategies.

As we see in the theoretical analysis as well as in the simulation study, the considered forward contracts influence significantly the volatility. Moreover, the impact is tuned by the number of suppliers in the industry. Comparing the cases with fixed and endogenous forward coverage, one can observe that the price volatility may be magnified by endogenous forward trading in a symmetric Cournot model with linear demand: the reason is that forward trading pushes spot prices closer to marginal costs, corresponding to a less elastic region of the demand function.

We plan to extend our analysis to more realistic models, such as the Supply Function Equilibrium model. Indeed, the Cournot model is a simplification of the actual decision-making by power suppliers - whose strategies concern price-quantity pairs. The works by Newbery (1998), Green (1999), and Borgosz-Koczwara and Wylomanska (2007) shall be the most useful starting points in that respect.

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VII. Biographies

Sandro Sapio (b. 1978) received the undergraduate (4-year) degree in International Economics from the University of Napoli Parthenope (2001) and the PhD degree in Economics and Management from the Sant’Anna School of Advanced Studies, Pisa (2005).

Since 2007 he is Assistant Professor at the Department of Economic Studies, University of Napoli Parthenope, where he teaches courses in Microeconomics, Industrial Organization, and Public Economics.

His topics of research include the empirical and theoretical analysis of electricity markets, and the empirical analysis of sectoral growth.

Agnieszka Wylomanska (b. 1978) since 2007 is Assistant Professor at the Institute of Mathematics and Computer Science at Wroclaw University of Technology (WUT). She graduated from WUT and received the Ph.D. degree in mathematics from Institute of Mathematics and Computer Science (WUT) in 2006.

Her area of interest are power system analysis, competitive electricity market, application of game theory to power market and time series analysis.

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